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A Detailed Stepwise Procedure to Perform Isogeometric Analysis of a Two Dimensional Continuum Plate Structure-II

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Abstract
Isogeometric analysis (IGA) is the future of analysis in structural mechanics where in the geometry of the structure can be precisely represented. The integration of CAD and IGA has enabled us to design the most complex geometry as well. The main focus of the paper is to present a detailed stepwise procedure to perform isogeometric analysis of continuum plate structures subjected to in-plane loading. In this paper, a simple example of a plate structure is taken and the NURBS basis functions were derived. The stiffness matrix was derived and the nodal displacements were determined. The plate structure is analyzed using MARC Mentat®, a standard finite element package, and the results show that the nodal displacements obtained are similar using both of these methods. The code is written in C++ to perform the isogeometric analysis of plate structures.

Keywords: *Isogeometric analysis, plate, continuum, NURBS, structural mechanics*

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INTRODUCTION

Isogeometric analysis helps us to integrate CAD and FEA, and precisely represent the geometry of the structure. The gap has been bridged between CAD and FEA and improved the engineering process. The computational geometry uses NURBS as the basis in engineering design. Recent trends in engineering analysis and high performance computing are also demanding greater precision and tighter integration of the overall modeling-analysis process [1]. The continuum structures are very widely used in the engineering design. The main focus of this study is to detail on the procedure to analyse the continuum plate structure using IGA. A simple example is considered here, a plate structure carrying in-plane loading is analyzed. Literature review is presented in the next part of the paper, the objectives and the scope of the study are presented, the theory required to perform the analysis is discussed after that. Then, the methodology followed to complete this study is discussed and the analysis is done, the results are presented and finally, the last part concludes this study with a brief on future study.

LITERATURE REVIEW

There are few papers on isogeometric analysis of plate structures. Luis, in his paper applied isogeometric analysis to solve the structural engineering problems in vibration analysis and geometric non-linearity [2]. Mit, in his summer internship, has applied isogoemetric analysis to analyze a few structures [3]. In his paper, he analyzed a few basic problems such as two dimensional plates and three dimensional structures using IGA [4]. Gondegaon, in his paper solved the plate problems and performed vibration analysis for one dimensional bar and beam problems and two dimensional plate problems as well [5]. Hartman applied isogeometric analysis in LS-DYNA and found that the results were better than FEA [6]. Hassani applied the isogeometric analysis to perform optimization using solid isotropic material with penalization to a few problems using the optimality criteria [7]. Lee, in his paper on optimum structural design, applied isogeometric analysis to solve a few problems in structural engineering [8]. Nagy *et al*. performed variational formulation of stress constraints in the isogeometric design of structures [9].

OBJECTIVES OF THE STUDY

1. To present a step-wise illustrative procedure to perform isogeometric analysis of a plate structure.

Scope of the Study

1. The study is limited to linear static analysis and Hooke's law is valid.
2. Buckling analysis is not included in the study.

THEORETICAL BACKGROUND

In this paper, the basic theory is discussed in this section. The NURBS basis functions and the parent to parametric mapping are discussed. The strain displacement matrix is presented and then the stiffness matrix is formed. In this paper an example of a two dimensional plate continuum analyzed using isogeometric analysis is also presented. The NURBS basis functions are used and are discussed first. The stiffness matrix is derived in a stepwise manner. The solution for the displacement vector at each node is compared with the results from the standard finite element analysis. The results show that the nodal displacements are in good agreement with the results obtained from IGA and the nodal displacements using standard FEA.

Basis Functions [5]

The basis functions are given by:

$$N_{i,0}(\xi) = \begin{Bmatrix} 1 & if\ \xi_i \le \xi < \xi_{i+1} \\ 0 & otherwise \end{Bmatrix} \tag{1}$$

For p=1, 2, 3, …. They are defined by:

$$N_{i,p}(\xi) = \frac{\xi-\xi_i}{\xi_{i+p}-\xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1}-\xi}{\xi_{i+p+1}-\xi_{i+1}} N_{i+1,p-1}(\xi) \tag{2}$$

This is referred to as the Cox-de Boor recursion formula.

Derivatives of B-Spline Basis Functions

The derivatives of the basis functions are given by:

$$\frac{d}{dx} N_{i,p}(\xi) = \frac{p}{\xi_{i+p}-\xi_i} N_{i,p-1}(\xi) - \frac{p}{\xi_{i+p+1}-\xi_{i+1}} N_{i+1,p-1}(\xi) \tag{3}$$

Generalized to Higher Order Derivatives [1]

The generalized higher order derivatives of the basis functions is given by:

$$\frac{d^k}{d^k\xi} N_{i,p}(\xi) = \frac{p}{\xi_{i+p}-\xi_i}\left(\frac{d^{k-1}}{d^{k-1}\xi} N_{i,p-1}(\xi)\right) - \frac{p}{\xi_{i+p+1}-\xi_{i+1}}\left(\frac{d^{k-1}}{d^{k-1}\xi} N_{i+1,p-1}(\xi)\right) \tag{4}$$

B-Spline Curves

The B-spline curve is given by:

$$C(\xi) = \sum_{i=1}^{n} N_{i,p}(\xi) B_i \tag{5}$$

B-Spline Surfaces

B-spline surfaces are given by:

$$S(\xi,\eta) = \sum_{i=1}^{n}\sum_{j=1}^{m} N_{i,p}(\xi) M_{j,q}(\eta)\, B_{i,j} \tag{6}$$

B-Spline Solids

B-Spline solids are given by:

$$S(\xi,\eta,\zeta) = \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{l} N_{i,p}(\xi) M_{j,q}(\eta) L_{k,r}(\zeta)\, B_{i,j,k} \tag{7}$$

NURBS Basis Function

With a given projective B-spline curve and its associated projective control points in hand, the control points for the NURBS curve are obtained by using the following relations:

$$(B_i)_j = \frac{(B_i^w)_j}{w_i}\ j = 1,2\,\ldots, d \tag{8}$$

$$w_i = (B_i^w)_{jd+1}$$

NURBS basis is given by:

For NURBS Curve

The NURBS curve is given by:

$$R_i^p(\xi) = \frac{N_{i,p}(\xi) w_i}{\sum_{i=1}^{n} N_{i,p}(\xi) w_i} \tag{9}$$

$$C(\xi) = \sum_{i=1}^{n} R_i^p(\xi) B_i$$

This is identical to the B-Splines.

For NURBS Surfaces

The NURBS surfaces are given by:

$$R_{i,j}^{p,q}(\xi,\eta) = \frac{N_{i,p}(\xi) M_{j,q}(\eta) w_{i,j}}{\sum_{i=1}^{n}\sum_{j=1}^{m} N_{i,p}(\xi) M_{j,q}(\eta) w_{i,j}} \tag{10}$$

For NURBS Solids

The NURBS solids are given by:

$$R_{i,j,k}^{p,q,r}(\xi,\eta,\zeta) = \frac{N_{i,p}(\xi) M_{j,q}(\eta) L_{k,r}(\zeta) w_{i,j,k}}{\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{l} N_{i,p}(\xi) M_{j,q}(\eta) L_{k,r}(\zeta) w_{i,j,k}} \tag{11}$$

Derivatives of NURBS

Apply the quotient rule, the derivatives of NURBS are given by:

$$\frac{d}{d\xi} R_i^p(\xi) = w_i \frac{W(\xi) N_{i,p}'(\xi) - W'(\xi) N_{i,p}(\xi)}{(W(\xi))^2} \tag{12}$$

$$where\ N_{i,p}'(\xi) = \frac{d}{d\xi} N_{i,p}(\xi)\ and\ W'(\xi) = \sum_{i=1}^{n} N_{i,p}'(\xi) w_i \tag{13}$$

***For Higher Order Derivatives of NURBS Basis Functions* [1]**

The higher order derivatives of NURBS basis functions are given by:

$$A_i^{(k)}(\xi) = w_i \frac{d^k}{d\xi^k} N_{i,p}(\xi)\ , (no\ sum\ on\ i) \tag{14}$$

We do not sum on the repeated index, and let,

$$W^{(k)}(\xi) = \frac{d^k}{d\xi^k} W(\xi)$$

Higher order derivatives can be expressed in terms of the lower order derivatives as:

$$\frac{d^k}{d\xi^k} R_i^p(\xi) = \frac{A_i^{(k)}(\xi) - \sum_{j=1}^{k} \binom{k}{j} W^{(j)}(\xi) \frac{d^{(k-j)}}{d\xi^{(k-j)}} R_i^p(\xi)}{W(\xi)} \tag{15}$$

$$where \binom{k}{j} = \frac{k!}{j!\,(k-j)!}$$

Parametric to Parent Mapping

The parametric to parent mapping is given by:

$$\xi = \frac{1}{2}\left[(\xi_{i+1} - \xi_i)\hat{\xi} + (\xi_{i+1} - \xi_i)\right] \tag{16}$$

$$\eta = \frac{1}{2}\left[(\eta_{i+1} - \eta_i)\hat{\eta} + (\eta_{i+1} - \eta_i)\right]$$

The Jacobian is given by:

$$J_{\bar{\xi},\bar{\eta}} = \frac{1}{4}(\xi_{i+1} - \xi_i)(\eta_{i+1} - \eta_i) \tag{17}$$

***Parametric Space to Physical Space* [5]**

The parametric space to physical space is given by:

$$X = N_1M_1X_1 + N_2M_1X_2 + N_2M_2X_3 + N_1M_2X_4$$

$$Y = N_1M_1Y_1 + N_2M_1Y_2 + N_2M_2Y_3 + N_1M_2Y_4$$

$$\begin{bmatrix}\frac{\partial N}{\partial \xi}\\ \frac{\partial N}{\partial \eta}\end{bmatrix} = \begin{bmatrix}\frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi}\\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta}\end{bmatrix}\begin{bmatrix}\frac{\partial N}{\partial x}\\ \frac{\partial N}{\partial y}\end{bmatrix} \tag{18}$$

Where,

$$\frac{\partial x}{\partial \xi} = \frac{\partial}{\partial \xi}[N_1M_1N_2M_1N_2M_2N_1M_2][x_1x_2x_3x_4]^T$$
$$\frac{\partial x}{\partial \eta} = \frac{\partial}{\partial \eta}[N_1M_1N_2M_1N_2M_2N_1M_2][x_1x_2x_3x_4]^T$$
$$\frac{\partial y}{\partial \xi} = \frac{\partial}{\partial \xi}[N_1M_1N_2M_1N_2M_2N_1M_2][y_1y_2y_3y_4]^T$$
$$\frac{\partial y}{\partial \eta} = \frac{\partial}{\partial \eta}[N_1M_1N_2M_1N_2M_2N_1M_2][y_1y_2y_3y_4]^T$$

Strain Displacement Matrix

The strain displacement matrix is given by:

$$B = \begin{bmatrix}\frac{\partial N}{\partial x} & 0\\ 0 & \frac{\partial N}{\partial y}\\ \frac{\partial N}{\partial y} & \frac{\partial N}{\partial x}\end{bmatrix} \tag{19}$$

The strain vector is given by:

$$\epsilon = \begin{bmatrix}\frac{\partial u}{\partial x}\\ \frac{\partial v}{\partial y}\\ \frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\end{bmatrix}$$

Where,

$$\begin{bmatrix}\frac{\partial u}{\partial x}\\ \frac{\partial u}{\partial y}\end{bmatrix} = \frac{1}{|J|}\begin{bmatrix}J_{22} & -J_{12}\\ -J_{21} & J_{11}\end{bmatrix}\begin{bmatrix}\frac{\partial u}{\partial \xi}\\ \frac{\partial u}{\partial \eta}\end{bmatrix} \tag{20}$$

The strain is given by:

$$\epsilon = AG = \frac{1}{|J|}\begin{bmatrix}J_{22} & -J_{12} & 0 & 0\\ 0 & 0 & -J_{21} & J_{11}\\ -J_{21} & J_{11} & J_{22} & -J_{12}\end{bmatrix}\begin{bmatrix}\frac{\partial u}{\partial \xi}\\ \frac{\partial u}{\partial \eta}\\ \frac{\partial v}{\partial \xi}\\ \frac{\partial v}{\partial \eta}\end{bmatrix}$$

***For Element 1* [10]**

$$\begin{bmatrix}\frac{\partial u}{\partial \xi}\\ \frac{\partial u}{\partial \eta}\\ \frac{\partial v}{\partial \xi}\\ \frac{\partial v}{\partial \eta}\end{bmatrix} = \begin{bmatrix}9\eta-3 & 0 & 3-9\eta & 0 & 9\eta & 0 & -9\eta & 0\\ 9\xi-3 & 0 & -9\xi & 0 & 9\xi & 0 & 3-9\xi & 0\\ 0 & 9\eta-3 & 0 & 3-9\eta & 0 & 9\eta & 0 & -9\eta\\ 0 & 9\xi-3 & 0 & -9\xi & 0 & 9\xi & 0 & 3-9\xi\end{bmatrix}\begin{bmatrix}u_1\\ u_2\\ u_3\\ u_4\\ u_5\\ u_6\\ u_7\\ u_8\end{bmatrix}$$

Plane Stress

The elasticity matrix for the material in plane stress condition is given by:

$$D = \frac{E}{(1-\upsilon^2)}\begin{bmatrix} 1 & \upsilon & 0 \\ \upsilon & 1 & 0 \\ 0 & 0 & (1-\upsilon)/2 \end{bmatrix} \tag{22}$$

Plane Strain

The elasticity matrix for the material in plane strain condition is given by:

$$D = \frac{E}{(1+\upsilon)(1-2\upsilon)}\begin{bmatrix} 1-\upsilon & \upsilon & 0 \\ \upsilon & 1-\upsilon & 0 \\ 0 & 0 & \left(\frac{1}{2}\right)-\upsilon \end{bmatrix}$$

***Stiffness Matrix* [11]**

The stiffness matrix is given by:

$$k = t\int_{-1}^{1}\int_{-1}^{1} B^T DB \left|J_{\xi,\eta}\right| d\xi\, d\eta \left|J_{\bar{\xi},\bar{\eta}}\right| weight \tag{23}$$

Gauss Quadrature

The Gauss quadrature points are given by:

$$\xi = \pm\frac{1}{\sqrt{3}}\ and\ \eta = \pm\frac{1}{\sqrt{3}}$$

Traction

$$\int u^T T = [u\ v]^T \begin{bmatrix} T_x \\ T_y \end{bmatrix} \left|J_{\xi,\eta}\right| d\xi\, d\eta \left|J_{\bar{\xi},\bar{\eta}}\right| weight$$

The traction is given by:

$$[u\ v]^T \begin{bmatrix} T_x \\ T_y \end{bmatrix} =$$

$$\begin{bmatrix} N_1M_1 & 0 \\ 0 & N_1M_1 \\ N_2M_1 & 0 \\ 0 & N_2M_1 \\ N_2M_2 & 0 \\ 0 & N_2M_2 \\ N_1M_2 & 0 \\ 0 & N_1M_2 \end{bmatrix}_{8x2} \begin{bmatrix} N_1M_1 & 0 & N_2M_1 & 0 & N_2M_2 & 0 & N_1M_2 & 0 \\ 0 & N_1M_1 & 0 & N_2M_1 & 0 & N_2M_2 & 0 & N_1M_2 \end{bmatrix}_{2x8} \begin{bmatrix} T_{x1} \\ T_{y1} \\ T_{x2} \\ T_{y2} \\ T_{x3} \\ T_{y3} \\ T_{x4} \\ T_{y4} \end{bmatrix}_{8x1} \tag{24}$$

Algorithm to Perform the IGA Analysis

The algorithm to perform the isogeometric analysis of a two dimensional plate structure carrying in-plane loading [3]:

1. Determine NURBS coordinates $(\xi_i, \xi_{i+}) * (\eta_j, \eta_{j+1})$ using elRangeU and elRangeV.
2. Store the connectivity of the element in an array names sctrB (of size nn).
3. Define strain displacement matrix B of size (1, 2*nn).
4. Set k_e=0.
5. Loop over Gauss points (GPs) $\{\xi_j', \omega_j'\}$ j=1, 2, …, n_{gp} where, ngp is the number of gauss points.

a) Compute parametric coordinate ξ corresponding to ξ_j'.

b) Compute $\left|J_{\xi'}\right|$ corresponding to the equations.

c) Compute the derivatives of the shape functions $R_{w\xi}^e$ and $R_{w\eta}^e$ at point ξ, η.

d) Compute J_ξ using control points (sctr(:,e)) $R_{w\xi}^e$ and $R_{w\eta}^e$.

e) Find J_ξ^{-1} and determinant$\left|J_\xi\right|$.

f) Compute the shape function derivatives$R_x = J_\xi^{-1}[R_{,\xi}^T R_{,\eta}^T]$.

g) Use R_x to build the strain displacement matrix B.

h) $k_e = k_e + B^T DB|J_{\xi'}||J_{\eta'}|\omega_j'$.

6. End loop on gauss points.
7. Assemble k_e into global stiffness matrix K^G.
8. End loop over all the elements.

METHODOLOGY

Although the literature available on isogeometric analysis is not very exhaustive, the analysis is done in a step wise manner. The existing literature is reviewed first, and the plate problem is chosen to present the isogeometric analysis of a plate structure in a step-wise illustrative approach. The basis functions were developed. The strain-displacement matrix and the stiffness matrix, force vector are assembled. The nodal displacements were calculated. The flowchart in Figure 1 shows the approach followed to complete this study.

ANALYSIS

The given domain is a plate structure having dimensions 30 mm×30 mm. The domain is discretized into nine first order four noded quadrilateral elements, each element having a dimension of 10 mm×10 mm as shown in the Figure 2a. The plate carries a load as shown in the Figure 2b. The right side edge is fixed, and the node 1 and the node 10 carry a roller support as shown in the Figure 2c. The knot vector is as shown in the Figure 2c. The modulus of elasticity is 2×10^5 N/mm^2 and the Poisson's ratio is 0.3. The element node connectivity is as shown in Table 1. The basis functions are as shown in the Table 2. The control points and knots are as shown in the Table 3.

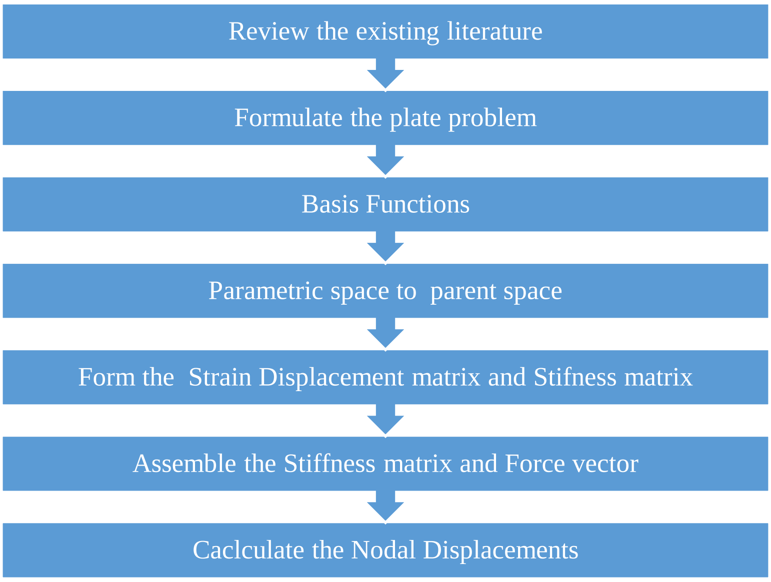
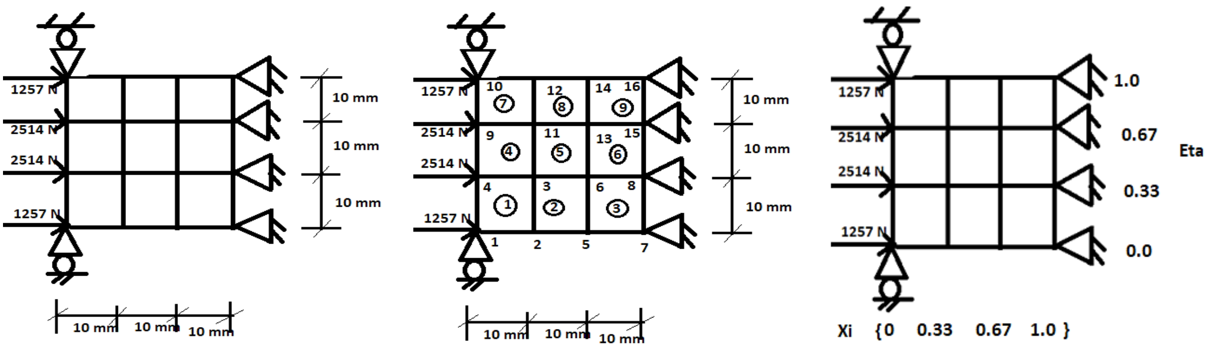


Fig. 1: *Flowchart Shows the Approach to Conduct This Study.*



(a) Plate (b) Meshing (c) Knot Vector

Fig. 2: *Showing the Two Dimensional Rectangular Domain.*

Table 1: *Showing the Element Node Connectivity Table.*

Element e	Node1	Node2	Node3	Node4
1	1	2	3	4
2	2	5	6	3
3	5	7	8	6
4	4	3	11	9
5	3	6	13	11
6	6	8	15	13
7	9	11	12	10
8	11	13	14	12
9	13	15	16	14

Table 2: *Showing the Basis Functions.*

Element 1 $\xi = 0\ and\ \xi = 0.33$ $\eta = 0\ and\ \eta = 0.33$	Element 2 $\xi = 0.33\ and\ \xi = 0.67$ $\eta = 0\ and\ \eta = 0.33$	Element 3 $\xi = 0.67\ and\ \xi = 1$ $\eta = 0\ and\ \eta = 0.33$
Element 4 $\xi = 0\ and\ \xi = 0.33$ $\eta = 0.33\ and\ \eta = 0.67$	Element 5 $\xi = 0.33\ and\ \xi = 0.67$ $\eta = 0.33\ and\ \eta = 0.67$	Element 6 $\xi = 0.67\ and\ \xi = 1$ $\eta = 0.33\ and\ \eta = 0.67$
Element 7 $\xi = 0\ and\ \xi = 0.33$ $\eta = 0.67\ and\ \eta = 1$	Element 8 $\xi = 0.33\ and\ \xi = 0.67$ $\eta = 0.67\ and\ \eta = 1$	Element 9 $\xi = 0.67\ and\ \xi = 1$ $\eta = 0.67\ and\ \eta = 1$

Table 3: *Showing the Node Coordinates/Control Points and Knots.*

Node	Control	Knot
1	(0, 0)	(0, 0)
2	(10, 0)	(0.33, 0)
3	(20, 0)	(0.67, 0)
4	(30, 0)	(1, 0)
5	(0, 10)	(0, 0.33)
6	(10, 10)	(0.33, 0.33)
7	(20, 10)	(0.67, 0.33)
8	(30, 10)	(1, 0.33)
9	(0, 20)	(0, 0.67)
10	(10, 20)	(0.33, 0.67)
11	(20, 20)	(0.67, 0.67)
12	(30, 20)	(1, 0.67)
13	(0, 30)	(0, 1)
14	(10, 30)	(0.33, 1)
15	(20, 30)	(0.67, 1)
16	(30, 30)	(1, 1)

Basis Functions

$$N_{i,p}(\xi) = \frac{\xi \quad - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi)$$

$$N_{i,0}(\xi) = \begin{Bmatrix} 1 \ \ if\ \xi_i \leq \ \xi < \xi_{i+1} \\ 0 \qquad otherwise \end{Bmatrix}$$

Important properties of the basis functions:
1. They constitute a partition of unity$\sum_{i=1}^{n} N_{i,p}(\xi) = 1$.
2. The support for basis function is compact within the interval of $[\xi_i\ ,\xi_{i+p+1}]$.
3. The basis functions are positive$\forall\ \xi_i$.

Knot Vector

A knot vector in one dimension is a set of coordinates in the parametric space.

$\Xi = \{\xi_1\xi_2 \dots \dots \dots \xi_{n+p-1}\xi_{n+p}\xi_{n+p+1}\} = \{0, 0, 0, 1/3, 2/3, 1, 1, 1\}$

Where, ξ_i is the i[th] knot in the Knot vector and p is the order of polynomial and n is the number of basis functions which comprise the B-Spline/NURBS, given weight is equal to one.

Derive the NURBS Basis Function for Element 1

For i=1 and p=1

$$N_{1,1} = \frac{\xi-\xi_1}{\xi_2-\xi_1}N_{1,0} + \frac{\xi_3-\xi}{\xi_3-\xi_2}N_{2,0} = \frac{\xi-0}{0-0}N_{1,0} + \frac{0-\xi}{0-0}N_{2,0} = 0$$

For i=2 and p=1

$$N_{2,1} = \frac{\xi-\xi_2}{\xi_3-\xi_2}N_{2,0} + \frac{\xi_4-\xi}{\xi_4-\xi_3}N_{3,0} = \frac{\xi-0}{0-0}N_{2,0} + \frac{0.3333-\xi}{0.3333-0}N_{3,0} = (1-3\xi)\ N_{3,0}$$

For i=3 and p=1

$$N_{3,1} = \frac{\xi-\xi_3}{\xi_4-\xi_3}N_{3,0} + \frac{\xi_5-\xi}{\xi_5-\xi_4}N_{4,0} = \frac{\xi-0}{1/3-0}N_{3,0} + \frac{0.66666-\xi}{0.6666-0.3333}N_{4,0}$$
$$= 3\xi N_{3,0} + (2-3\xi)N_{4,0}$$

For i=4 and p=1

$$N_{4,1} = \frac{\xi-\xi_4}{\xi_5-\xi_4}N_{4,0} + \frac{\xi_6-\xi}{\xi_6-\xi_5}N_{5,0} = \frac{\xi-1/3}{\frac{2}{3}-1/3}N_{4,0} + \frac{1-\xi}{1-2/3}N_{5,0} = (3\xi-1)N_{4,0} + (3-3\xi)N_{5,0}$$

For i=5 and p=1

$$N_{5,1} = \frac{\xi-\xi_5}{\xi_6-\xi_5}N_{5,0} + \frac{\xi_7-\xi}{\xi_7-\xi_6}N_{6,0} = \frac{\xi-2/3}{1-\frac{2}{3}}N_{5,0} + \frac{1-\xi}{1-1}N_{6,0} = (3\xi-2)N_{5,0}$$

For i=6 and p=1

$$N_{6,1} = \frac{\xi-\xi_6}{\xi_7-\xi_6}N_{6,0} + \frac{\xi_8-\xi}{\xi_8-\xi_7}N_{7,0} = \frac{\xi-1}{1-1}N_{6,0} + \frac{1-\xi}{1-1}N_{7,0} = 0$$

Gauss Points of Integration

For element 1

$$\xi = 0\ and\ \xi = \frac{1}{3};\ \eta = 0\ and\ \eta = \frac{1}{3}$$

The Jacobian is $J_{\bar{\xi},\bar{\eta}} = \frac{1}{4}(\xi_{i+1}-\xi_i)(\eta_{i+1}-\eta_i) = \frac{1}{4} * \left(\frac{1}{3}-0\right) * \left(\frac{1}{3}-0\right) = \frac{1}{36}$

Parent element Gauss Points of Integration are $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$

Parametric space $= \frac{1}{2}\ [(\xi_{i+1}-\xi_i)\hat{\xi} + (\xi_{i+1}-\xi_i)]$

$$= \frac{1}{2}\left[\left(\frac{1}{3}-0\right)\left(\frac{1}{\sqrt{3}}\right) + \left(\frac{1}{3}-0\right)\right] = 0.262891$$
$$= \frac{1}{2}\left[\left(\frac{1}{3}-0\right)\left(\frac{-1}{\sqrt{3}}\right) + \left(\frac{1}{3}-0\right)\right] = 0.070441$$

The Parametric Space to Physical Space:
Gauss points of Integration are:
(0.262891,0.262891) (0.262891,0.070441) (0.070441,0.262891) (0.070441,0.070441)

Isoparametric Elements
The displacement in terms of nodal displacements is given by:
$U=N_1M_1U_1+N_2M_1U_3+N_2M_2U_5+N_1M_2U_7$
$V=N_1M_1U_2+N_2M_1U_4+N_2M_2U_6+N_1M_2U_8$

The co-ordinates in terms of the nodal co-ordinates are given by:
$X=N_1M_1X_1+N_2M_1X_2+N_2M_2X_3+N_1M_2X_4$
$Y=N_1M_1Y_1+N_2M_1Y_2+N_2M_2Y_3+N_1M_2Y_4$

Derivative of NURBS Basis Function
The basis functions are $N_1 = (1-3\xi) and\ N_2 = 3\xi\ and\ M_1 = (1-3\eta) and\ M_2 = 3\eta$

$$\frac{\partial(N_1M_1)}{\partial\xi} = (-3)(1-3\eta)\ and\ \frac{\partial(N_2M_1)}{\partial\xi} = (3)(1-3\eta)$$

$$\frac{\partial(N_2M_2)}{\partial\xi} = (3)(3\eta)\ and\ \frac{\partial(N_1M_2)}{\partial\xi} = (-3)(3\eta)$$

$$\frac{\partial(N_1M_1)}{\partial\eta} = (-3)(1-3\xi)\ and\ \frac{\partial(N_2M_1)}{\partial\eta} = (3\xi)(-3)$$

$$\frac{\partial(N_2M_2)}{\partial\eta} = (3)(3\xi)\ and\ \frac{\partial(N_1M_2)}{\partial\eta} = (1-3\xi)(3)$$

The Jacobian in parent space is given by: $\begin{bmatrix} \frac{\partial x}{\partial\xi} & \frac{\partial y}{\partial\xi} \\ \frac{\partial x}{\partial\eta} & \frac{\partial y}{\partial\eta} \end{bmatrix} = \begin{bmatrix} 30 & 0 \\ 0 & 30 \end{bmatrix}$

The co-ordinates are given by: $X_1=0$, $X_2=10$, $X_3=10$, $X_4=0$; $Y_1=0$, $Y_2=0$, $Y_3=10$, $Y_4=10$.

Derive the Stiffness Matrix at Integration Point (0.262891, 0.070441)
$At\ Integration\ point 1$ (0.262891, 0.070441)

$$\frac{\partial x}{\partial\xi} = \frac{\partial}{\partial\xi}(N_1M_1X_1 + N_2M_1X_2 + N_2M_2X_3 + N_1M_2X_4)$$

$$\frac{\partial x}{\partial\xi} = (-3)(1-3*0.070441)(0) + (3)(1-3*0.070441)*10 + (3)(3*0.070441)*10 + (3)(-3*0.070441)*(0) = 30$$

$$\frac{\partial y}{\partial\xi} = \frac{\partial}{\partial\xi}(N_1M_1Y_1 + N_2M_1Y_2 + N_2M_2Y_3 + N_1M_2Y_4)$$

$$\frac{\partial y}{\partial\xi} = (-3)(1-3*0.070441)(0) + (3)(1-3*0.070441)*0 + (3)(3*0.070441)*10 + (3)(-3*0.070441)*(10) = 0$$

$$\frac{\partial x}{\partial\eta} = \frac{\partial}{\partial\eta}(N_1M_1X_1 + N_2M_1X_2 + N_2M_2X_3 + N_1M_2X_4)$$

$$\frac{\partial x}{\partial \eta} = (-3)(1-3*0.262891)(0) + (3*0.262891)(-3)(10) + (3)(3*0.262891)*10 + (1-3*0.262891)(3)*0 = 0$$

$$\frac{\partial y}{\partial \eta} = \frac{\partial}{\partial \eta}(N_1M_1Y_1 \ + \ N_2M_1Y_2 \ + \ N_2M_2Y_3 \ + \ N_1M_2Y_4)$$

$$\frac{\partial y}{\partial \eta} = (-3)(1-3*0.262891)(0) + (3*0.262891)(-3)(0) + (3)(3*0.262891)*10 + (1-3*0.262891)(3)*10 = 30$$

The strain displacement matrix is $B = \begin{bmatrix} \frac{\partial N}{\partial x} & 0 \\ 0 & \frac{\partial N}{\partial y} \\ \frac{\partial N}{\partial y} & \frac{\partial N}{\partial x} \end{bmatrix}$

$\frac{\partial N}{\partial \xi} = \frac{\partial N}{\partial x}\frac{\partial x}{\partial \xi} + \frac{\partial N}{\partial y}\frac{\partial y}{\partial \xi}$ $\frac{\partial N}{\partial \eta} = \frac{\partial N}{\partial x}\frac{\partial x}{\partial \eta} + \frac{\partial N}{\partial y}\frac{\partial y}{\partial \eta}$

$$\begin{bmatrix} \frac{\partial N}{\partial x} \\ \frac{\partial N}{\partial y} \end{bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial N}{\partial \xi} \\ \frac{\partial N}{\partial \eta} \end{bmatrix}$$

Where $N\ is\ the\ basis\ function\ matrix = [N_1M_1 \ \ N_2M_1 \ N_2M_2N_1M_2]^T$

$$strain\ \varepsilon = \begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{bmatrix}$$

$$\begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \end{bmatrix} = \frac{1}{|J|}\begin{bmatrix} J_{22} & -J_{12} \\ -J_{21} & J_{11} \end{bmatrix} \begin{bmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \end{bmatrix}$$

$\epsilon = A\ G\ U = B\ U$

$$\epsilon = \frac{1}{|J|}\begin{bmatrix} J_{22} & -J_{12} & 0 & 0 \\ 0 & 0 & -J_{21} & J_{11} \\ -J_{21} & J_{11} & J_{22} & -J_{12} \end{bmatrix} \begin{bmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \\ \frac{\partial v}{\partial \xi} \\ \frac{\partial v}{\partial \eta} \end{bmatrix}$$

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$$\begin{bmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \\ \frac{\partial v}{\partial \xi} \\ \frac{\partial v}{\partial \eta} \end{bmatrix} = \begin{bmatrix} 9\eta-3 & 0 & 3-9\eta & 0 & 9\eta & 0 & -9\eta & 0 \\ 9\xi-3 & 0 & -9\xi & 0 & 9\xi & 0 & 3-9\xi & 0 \\ 0 & 9\eta-3 & 0 & 3-9\eta & 0 & 9\eta & 0 & -9\eta \\ 0 & 9\xi-3 & 0 & -9\xi & 0 & 9\xi & 0 & 3-9\xi \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \end{bmatrix}$$

B=AG

$$\frac{1}{900}\begin{bmatrix} 30 & 0 & 0 & 0 \\ 0 & 0 & 0 & 30 \\ 0 & 30 & 30 & 0 \end{bmatrix} \begin{bmatrix} 9*0.070441-3 & 0 & 3-9*0.070441 & 0 & 9*0.070441 & 0 & -9*0.070441 & 0 \\ 9*0.262891-3 & 0 & -9*0.262891 & 0 & 9*0.262891 & 0 & 3-9*0.262891 & 0 \\ 0 & 9*0.070441-3 & 0 & 3-9*0.070441 & 0 & 9*0.070441 & 0 & -9*0.070441 \\ 0 & 9*0.262891-3 & 0 & -9*0.262891 & 0 & 9*0.262891 & 0 & 3-9*0.262891 \end{bmatrix}$$

$$B = \frac{1}{900}\begin{bmatrix} -70.980 & 0 & 70.980 & 0 & 19.019 & 0 & -19.019 & 0 \\ 0 & -19.019 & 0 & -70.980 & 0 & 70.980 & 0 & 19.019 \\ -19.019 & -70.980 & -70.980 & 70.98 & 70.98 & 19.019 & 19.019 & -19.019 \end{bmatrix}$$

The stiffness matrix is given by: $\int_v B^T DB \ dv = t \iint_0^{1/3} B^T DB \left|J_{\xi,\eta}\right|\left|J_{\bar{\xi},\bar{\eta}}\right| d\xi \, d\eta * weight$
The numerical integration is performed using 2×2 Gauss integration.

Where, D is the constitutive matrix in plane stress condition. $\frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} = \frac{2\ 10^5}{1-0.3^2}\begin{bmatrix} 1 & 0.3 & 0 \\ 0.3 & 1 & 0 \\ 0 & 0 & 0.35 \end{bmatrix}$

The multiple = (1/36)
The value of $B^T DB$ is as shown in the Table 4.

Table 4: *Showing the $B^T DB$ Matrix for Element 1.*

1.26126e+006	214286	–1.11496e+006	253719	–445055	–400021	298753	–67984
214286	518956	331720	–100951	–457122	–445055	–88884.2	27049.6
–1.11496e+006	331720	1.66097e+006	–799725	–100951	253720	–445055	214286
253719	–100951	–799725	1.66097e+006	331720	–1.11496e+006	214286	–445055
–445055	–457122	–100951	331720	518956	214285	27049.9	–88884
–400021	–445055	253720	–1.11496e+006	214285	1.26126e+006	–67983.8	298753
298753	–88884.2	–445055	214286	27049.9	–67983.8	119252	–57417.6
–67984	27049.6	214286	–445055	–88884	298753	–57417.6	119252

Derive the Stiffness Matrix at Integration Point (0.262891, 0.262891)

$At\ Integration\ point 2\ (0.262891, 0.262891)$

$$\frac{\partial x}{\partial \xi} = \frac{\partial}{\partial \xi}(N_1 M_1 X_1 + N_2 M_1 X_2 + N_2 M_2 X_3 + N_1 M_2 X_4)$$

$$\frac{\partial x}{\partial \xi} = (-3)(1-3*0.262891)(0) + (3)(1-3*0.262891)*10 + (3)(3*0.262891)*10 + (3)(-3*0.262891)*(0) = 30$$

$$\frac{\partial y}{\partial \xi} = \frac{\partial}{\partial \xi}(N_1M_1Y_1 \ + \ N_2M_1Y_2 \ + \ N_2M_2Y_3 \ + \ N_1M_2Y_4)$$

$$\frac{\partial y}{\partial \xi} = (-3)(1-3*0.262891)(0) + \ (3)(1-3*0.262891)*0 + (3)(3*0.262891)*10 \\ + \ (3)(-3*0.262891)*(10) = \ 0$$

$$\frac{\partial x}{\partial \eta} = \frac{\partial}{\partial \eta}(N_1M_1X_1 \ + \ N_2M_1X_2 \ + \ N_2M_2X_3 \ + \ N_1M_2X_4)$$

$$\frac{\partial x}{\partial \eta} = (-3)(1-3*0.262891)(0) + (3*0.262891)(-3)(10) + (3)(3*0.262891)*10 \\ + (1-3*0.262891)(3)*0 = 0$$

$$\frac{\partial y}{\partial \eta} = \frac{\partial}{\partial \eta}(N_1M_1Y_1 \ + \ N_2M_1Y_2 \ + \ N_2M_2Y_3 \ + \ N_1M_2Y_4)$$

$$\frac{\partial y}{\partial \eta} = (-3)(1-3*0.262891)(0) + (3*0.262891)(-3)(0) + (3)(3*0.262891)*10 \\ + (1-3*0.262891)(3)*10 = 30$$

$$B = \frac{1}{900}\begin{bmatrix} -19019 & 0 & 19.0910 & 0 & 70.98 & 0 & -70.98 & 0 \\ 0 & -19.019 & 0 & -70.980 & 0 & 70.980 & 0 & 19.019 \\ -19.019 & -19.019 & -70.980 & 19.019 & 70.98 & 70.98 & 19.019 & -70.98 \end{bmatrix}$$

The value of B^TDB is as shown in the Table 5.

Table 5: *Showing the B^TDB Matrix.*

119253	57417.9	27049.1	67984.1	–445056	–214286	298754	88884
57417.9	119252	88884.5	298753	–214286	–445055	67983.7	27050
27049.1	88884.5	518957	–214286	–100950	–331719	–445056	457121
67984.1	298753	–214286	1.26126e+006	–253718	–1.11496e+006	400021	–445055
–445056	–214286	–100950	–253718	1.66097e+006	799724	–1.11496e+006	–331720
–214286	–445055	–331719	–1.11496e+006	799724	1.66097e+006	–253719	–100950
298754	67983.7	–445056	400021	–1.11496e+006	–253719	1.26126e+006	–214286
88884	27050	457121	–445055	–331720	–100950	–214286	518955

Derive the Stiffness Matrix at Integration Point (0.070441, 0.070441)

$At\ Integration\ point 3\ (\ 0.070441, 0.070441)$

$$\frac{\partial x}{\partial \xi} = \frac{\partial}{\partial \xi}(N_1M_1X_1 \ + \ N_2M_1X_2 \ + \ N_2M_2X_3 \ + \ N_1M_2X_4)$$

$$\frac{\partial x}{\partial \xi} = (-3)(1-3*\ 0.070441)(0) + \ (3)(1-3*\ 0.070441)*10 + (3)(3*\ 0.070441)*10 \\ + \ (3)(-3*0\ 0.070441)*(0) = \ 30$$

$$\frac{\partial y}{\partial \xi} = \frac{\partial}{\partial \xi}(N_1M_1Y_1 \ + \ N_2M_1Y_2 \ + \ N_2M_2Y_3 \ + \ N_1M_2Y_4)$$

$$\frac{\partial y}{\partial \xi} = (-3)(1-3*\ 0.070441)(0) + \ (3)(1-3*\ 0.070441)*0 + (3)(3*\ 0.070441)*10 \\ + \ (3)(-3*\ 0.070441)*(10) = \ 0$$

$$\frac{\partial x}{\partial \eta} = \frac{\partial}{\partial \eta}(N_1M_1X_1 \; + \; N_2M_1X_2 \; + \; N_2M_2X_3 \; + \; N_1M_2X_4)$$

$$\frac{\partial x}{\partial \eta} = (-3)(1-3*\ 0.070441)(0) + (3*\ 0.070441)(-3)(10) + (3)(3*\ 0.070441)*10 + (1-3*\ 0.070441)(3)*0 = 0$$

$$\frac{\partial y}{\partial \eta} = \frac{\partial}{\partial \eta}(N_1M_1Y_1 \; + \; N_2M_1Y_2 \; + \; N_2M_2Y_3 \; + \; N_1M_2Y_4)$$

$$\frac{\partial y}{\partial \eta} = (-3)(1-3*\ 0.070441)(0) + (3*\ 0.070441)(-3)(0) + (3)(3*\ 0.070441)*10 + (1-3*\ 0.070441)(3)*10 = 30$$

$$B = \frac{1}{900}\begin{bmatrix} -70.98 & 0 & 70.98 & 0 & 19.019 & 0 & -19.019 & 0 \\ 0 & -70.98 & 0 & -19.019 & 0 & 19.019 & 0 & 70.98 \\ -70.98 & -70.98 & -19.019 & 70.98 & 19.019 & 19.019 & 70.98 & -19.019 \end{bmatrix}$$

The value of B^TDB is as shown in the Table 6.

Table 6: *Showing the B^TDB Matrix.*

1.66097e+006	799725	–1.11496e+006	–331720	–445055	–214286	–100951	–253720
799725	1.66097e+006	–253719	–100951	–214286	–445055	–331720	–1.11496e+006
–1.11496e+006	–253719	1.26126e+006	–214286	298753	67983.9	–445055	400021
–331720	–100951	–214286	518956	88884.2	27049.6	457122	–445055
–445055	–214286	298753	88884.2	119252	57417.6	27049.9	67983.8
–214286	–445055	67983.9	27049.6	57417.6	119252	88884.1	298753
–100951	–331720	–445055	457122	27049.9	88884.1	518956	–214286
–253720	–1.11496e+006	400021	–445055	67983.8	298753	–214286	1.26126e+006

Derive the Stiffness Matrix at Integration Point (0.070441, 0.262891)

$At\ Integration\ point4$ (0.070441,0.262891)

$$\frac{\partial x}{\partial \xi} = \frac{\partial}{\partial \xi}(N_1M_1X_1 \; + \; N_2M_1X_2 \; + \; N_2M_2X_3 \; + \; N_1M_2X_4)$$

$$\frac{\partial x}{\partial \xi} = (-3)(1-3*0.262891)(0) + \ (3)(1-3*0.262891)*10 + (3)(3*0.262891)*10 + \ (3)(-3*0.262891)*(0) = \ 30$$

$$\frac{\partial y}{\partial \xi} = \frac{\partial}{\partial \xi}(N_1M_1Y_1 \; + \; N_2M_1Y_2 \; + \; N_2M_2Y_3 \; + \; N_1M_2Y_4)$$

$$\frac{\partial y}{\partial \xi} = (-3)(1-3*0.262891)(0) + \ (3)(1-3*0.262891)*0 + (3)(3*0.262891)*10 + \ (3)(-3*0.262891)*(10) = \ 0$$

$$\frac{\partial x}{\partial \eta} = \frac{\partial}{\partial \eta}(N_1M_1X_1 \; + \; N_2M_1X_2 \; + \; N_2M_2X_3 \; + \; N_1M_2X_4)$$

$$\frac{\partial x}{\partial \eta} = (-3)(1-3*0.070441)(0) + (3*0.070441)(-3)(10) + (3)(3*0.070441)*10 + (1-3*0.070441)(3)*0 = 0$$

$$\frac{\partial y}{\partial \eta} = \frac{\partial}{\partial \eta}(N_1M_1Y_1 \; + \; N_2M_1Y_2 \; + \; N_2M_2Y_3 \; + \; N_1M_2Y_4)$$

$$\frac{\partial y}{\partial \eta} = (-3)(1-3*0.070441)(0) + (3*0.070441)(-3)(0) + (3)(3*0.070441)*10 + (1-3*0.070441)(3)*10 = 30$$

$$B = \frac{1}{900}\begin{bmatrix} -19.019 & 0 & 19.019 & 0 & 70.98 & 0 & -70.98 & 0 \\ 0 & -70.98 & 0 & -19.019 & 0 & 19.019 & 0 & 70.98 \\ -70.98 & -19.019 & -19.019 & 19.019 & 19.019 & 70.98 & 70.98 & -70.98 \end{bmatrix}$$

The value of B^TDB is as shown in the Table 7.

Table 7: *Showing the B^TDB Matrix.*

518957	214287	27049.1	–88884.5	–445056	–457121	–100950	331719
214287	1.26126e+006	–67984.1	298753	–400021	–445055	253719	–1.11496e+006
27049.1	–67984.1	119253	–57417.9	298754	–88883.9	–445056	214286
–88884.5	298753	–57417.9	119252	–67983.7	27050	214286	–445055
–445056	–400021	298754	–67983.7	1.26126e+006	214285	–1.11496e+006	253719
–457121	–445055	–88883.9	27050	214285	518955	331720	–100950
–100950	253719	–445056	214286	–1.11496e+006	331720	1.66097e+006	–799724
331719	–1.11496e+006	214286	–445055	253719	–100950	–799724	1.66097e+006

The Stiffness Matrix

The stiffness matrix is as shown in the Table 8.

Table 8: *Showing the Stiffness Matrix for Element 1.* (1/36) *

3560440	1285716	–2175822	–98901.4	–1780222	–1285714	395606	98899
1285716	3560438	98901.4	395604	–1285715	–1780220	–98901.5	–2175820
–2175822	98901.4	3560440	–1285715	395606	–98899	–1780222	1285714
–98901.4	395604	–1285715	3560438	98902.5	–2175820	1285715	–1780220
–1780222	–1285715	395606	98902.5	3560438	1285712	–2175820	–98901.2
–1285714	–1780220	–98899	–2175820	1285712	3560437	98901.3	395606
395606	–98901.5	–1780222	1285715	–2175820	98901.3	3560438	–1285714
98899	–2175820	1285714	–1780220	–98901.2	395606	–1285714	3560437

Stiffness Matrix for Element 1

The stiffness matrix for the element 1 is as shown in the Table 9.

Table 9: *Showing the Stiffness Matrix for Element 1.*

98901.1	35714.3	–60439.6	–2747.26	–49450.5	–35714.2	10989	2747.21
35714.3	98901	2747.25	10989	–35714.2	–49450.5	–2747.29	–60439.5
–60439.6	2747.25	98901.1	–35714.3	10989	–2747.21	–49450.5	35714.2
–2747.26	10989	–35714.3	98901	2747.29	–60439.5	35714.2	–49450.5
–49450.5	–35714.2	10989	2747.29	98900.8	35714.2	–60439.3	–2747.24
–35714.2	–49450.5	–2747.21	–60439.5	35714.2	98900.9	2747.25	10989.1
10989	–2747.29	–49450.5	35714.2	–60439.3	2747.25	98900.8	–35714.2
2747.21	–60439.5	35714.2	–49450.5	–2747.24	10989.1	–35714.2	98900.9

Apply Boundary Conditions

The nodal displacements at nodes 7, 8, 15, 16 at supports is equal to zero. The degrees of freedom $U_{13}=U_{14}=U_{15}=U_{16}=U_{29}=U_{30}=U_{31}=U_{32}=0$.

The Y-displacement at node 1 and node 10 is equal to zero. $U_2=U_{20}=0$.

Reduced Global Stiffness Matrix

Table 10 shows the reduced stiffness matrix for the plate structure.

Size of the reduced stiffness matrix =16×2=32–2×4–1×2=22 rows and 22 columns.

Table 10: *Showing the Reduced Stiffness Matrix for the Plate.*

98901.1	–60439.6	–2747.26	–49450.5	–35714.2	10989	2747.21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
–60439.6	197802	0.0128571	21978.1	0.000988997	–49450.5	35714.2	–60439.6	–2747.26	–49450.5	–35714.3	0	0	0	0	0	0	0	0	0	0	0
–2747.26	0.0128571	197802	–0.000989023	–120879	35714.2	–49450.5	2747.25	10989	–35714.3	–49450.5	0	0	0	0	0	0	0	0	0	0	0
–49450.5	21978.1	–0.000989023	395604	–0.0128571	–120879	0.0144396	–49450.5	35714.3	–120879	–0.0134506	–49450.6	35714.3	0	21978	–6.92317e–006	0	0	–49450.6	–35714.3	0	0
–35714.2	0.000988997	–120879	–0.0128571	395604	–0.008011	21978	35714.2	–49450.5	0.00702196	21978	35714.3	–49450.6	0	–5.93394e–006	–120879	0	0	–35714.3	–49450.6	0	0
10989	–49450.5	35714.2	–120879	–0.008011	197802	0.142714	0	0	0	0	10989	2747.22	0	–49450.6	35714.3	0	0	0	0	0	0
2747.21	35714.2	–49450.5	0.0144396	21978	0.142714	197802	0	0	0	0	–2747.29	–60439.6	0	–35714.3	–49450.6	0	0	0	0	0	0
0	–60439.6	2747.25	–49450.5	35714.2	0	0	197802	0.00287146	21978.1	0.0382978	0	0	0	0	0	0	0	0	0	0	0
0	–2747.26	10989	35714.3	–49450.5	0	0	0.00287146	197802	0.0324164	–120879	0	0	0	0	0	0	0	0	0	0	0
0	–49450.5	–35714.3	–120879	0.00702196	0	0	21978.1	0.0324164	395604	–0.115714	0	0	0	–49450.6	35714.3	0	0	21978	0.0348363	0	0
0	–35714.3	–49450.5	–0.0134506	21978	0	0	0.0382978	–120879	–0.115714	395604	0	0	0	35714.3	–49450.6	0	0	0.035878	–120879	0	0
0	0	0	–49450.6	35714.3	10989	–2747.29	0	0	0	0	197802	1.71782e–005	10989	–120879	–0.00296835	–49450.5	–35714.3	0	0	0	0
0	0	0	35714.3	–49450.6	2747.22	–60439.6	0	0	0	0	1.71782e–005	197802	–2747.25	–0.00346022	21978.1	–35714.3	–49450.5	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	10989	–2747.25	98901.1	–49450.5	35714.3	–60439.6	2747.26	0	0	0	0
0	0	0	21978	–5.93	–494	–357	0	0	–4945	35714.3	–1208	–0.00	–49	395604	0.012882	21978	0.000983	–1208	–0.00	–494	–357

				394e –006	50.6	14.3			0.6		79	346022	450.5		9		077	79	346484	50.5	14.3
0	0	0	–6.92317e –006	–120879	–35714.3	–49450.6	0	0	35714.3	–49450.6	–0.00296835	21978.1	35714.3	0.0128829	395605	–0.000995934	–120879	–0.00296373	21978.1	–35714.3	–49450.5
0	0	0	0	0	0	0	0	0	0	0	–49450.5	–35714.3	–60439.6	21978	–0.000995934	197802	–0.0128571	–49450.5	35714.3	–60439.6	2747.26
0	0	0	0	0	0	0	0	0	0	0	–35714.3	–49450.5	2747.26	0.000983077	–120879	–0.0128571	197802	35714.3	–49450.5	–2747.24	10989
0	0	0	–49450.6	–35714.3	0	0	0	0	21978	0.035878	0	0	0	–120879	–0.00296373	–49450.5	35714.3	395604	–3.53029e –008	21978	0.0021989
0	0	0	–35714.3	–49450.6	0	0	0	0	0.0348363	–120879	0	0	0	–0.00346484	21978.1	35714.3	–49450.5	–3.53029e –008	395604	–0.0021989	–120879
0	0	0	0	0	0	0	0	0	0	0	0	0	0	–49450.5	–35714.3	–60439.6	–2747.24	21978	–0.0021989	197802	–0.0285857
0	0	0	0	0	0	0	0	0	0	0	0	0	0	–35714.3	–49450.5	2747.26	10989	0.0021989	–120879	–0.0285857	197802

Reduced Global Force Vector
The force vector is given by:
F11=1257 (Node 1 Dof 1),
F41=2514 (Node 4 Dof 1),
F91=2514 (Node 9 Dof 1),
F101=1257 (Node 10 Dof 1).

The magnitude is zero at all other nodes.

RESULTS AND DISCUSSION

The Table 11 shows the comparison of results of analysis using isogeometric NURBS basis functions and Finite element analysis using MARC Mentat®.

Table 11: *Showing the Comparison of Nodal Displacements Using IGA and FEA.*

NodeDof	IGA	FEA using MARC MENTAT®
Node1 Dof1	0.0346642	0.0346642
Node1 Dof2	0	0
21	0.023636	0.023636
22	–0.00565269	–0.00565269
31	0.0249371	0.0249371
32	–0.00180772	–0.00180772
41	0.0374289	0.0374289
42	–0.000373083	–0.000373079
51	0.0125645	0.0125645
52	–0.00530364	–0.00530364

61	0.0116714	0.0116714
62	–0.00162245	–0.00162245
71	0	0
72	0	–1.0e–12
81	0	0
82	0	–2.89e–13
91	0.0374289	0.0374289
92	0.000373079	0.000373079
101	0.0346642	0.0346642
102	0	9.083e–13
111	0.0249371	0.0249371
112	0.00180772	0.00180772
121	0.023636	0.023636
122	0.00565269	0.00565269
131	0.0116714	0.0116714
132	0.00162245	0.00162245
141	0.0125645	0.0125645
142	0.00530364	0.00530364
151	0	0
152	0	0
161	0	0
162	0	0

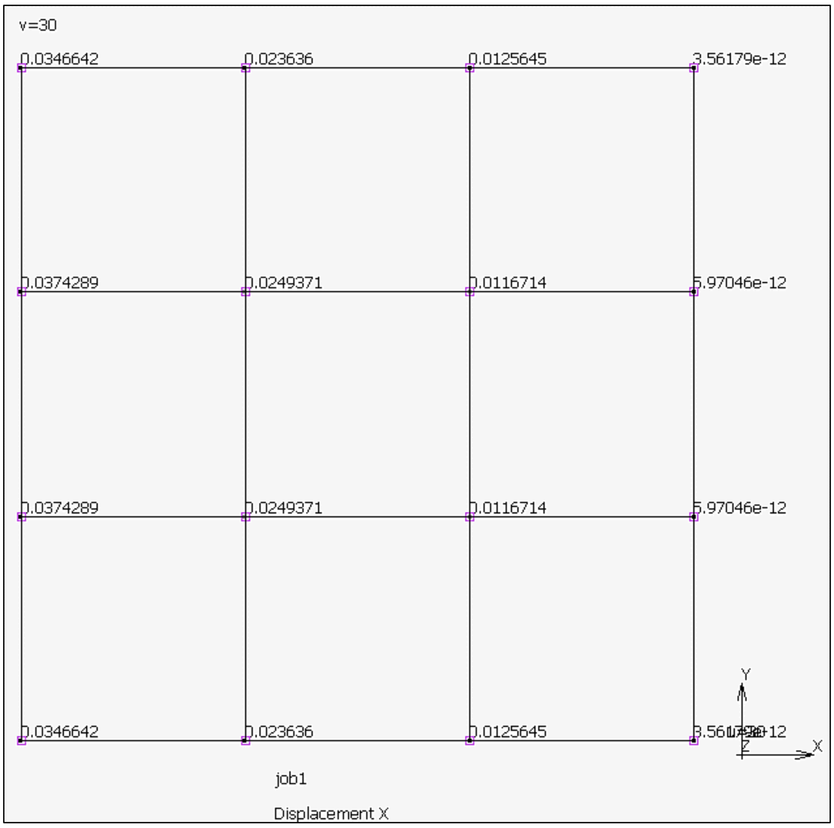

***Fig. 3:** Showing the Nodal X-Displacements Using MARC Mentat®.*

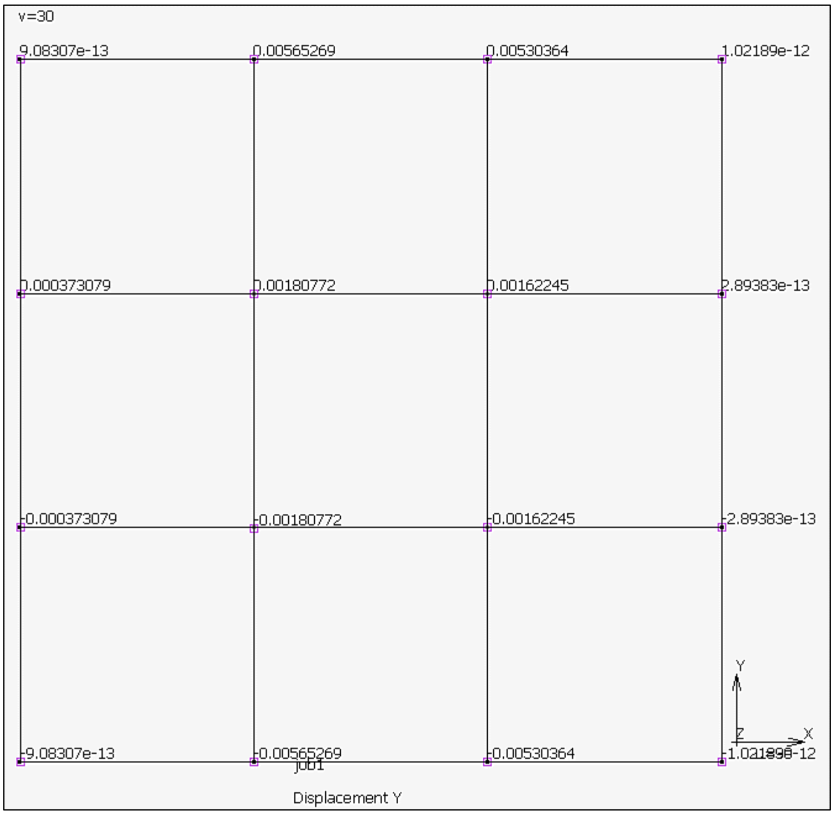

Fig. 4: *Showing the Nodal Y-Displacements Using MARC Mentat®.*

Finite Element Analysis using MARC Mentat®

The given plate domain is analysed using finite element analysis package MARC Mentat®. The loading, geometry, boundary conditions, and force are applied as per the problem. The domain is discretized using first order four node quadrilateral elements. The nodal displacements are calculated. The results show that the nodal displacements calculated using the isogeometric analysis are similar to the nodal displacements obtained using the finite element analysis as shown in the Table 11. Figure 3 shows the X-displacements at each node and Figure 4 shows the Y-displacements at each node using MARC Mentat®.

CONCLUSIONS

The literature review on isogeometric analysis of plate structures is done. There are few papers on isogeometric analysis and the problems and solutions discussed in these did not have a stepwise formulation and solution. The paper is meant to address this issue to present a stepwise formulation and which can also be used as a class room example. In this paper, the formulation is presented only for element 1 and similar steps can be followed for all the other elements in the domain. The nodal displacements were calculated at each node using isogeometric analysis and standard finite element analysis using MARC Mentat®. The results clearly show that the nodal displacements obtained were similar using both of these methods.

FUTURE STUDY

Isogeometric anlaysis is the future of the structural analysis in which the geometry of the complex domains can be exactly represented. The drawback of finite element analysis to represent the domain precisely has been addressed using CAD and IGA. This can be applied to other problems in mechanics such as shells, topology optimization and fracture mechanics as well.

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